APPENDIX A

PHYSICS OF METEOROLOGICAL RADARS

A.1 <u>Detection of Precipitation</u>. Consider a monostatic radar such as the WSR-88D with peak transmitted power, P_t, and antenna gain, G, illuminating a single target cross section, σ_b, at range, r.

The incident power density, P_i, assuming no intervening losses, is:

$$P_i = P_t G / 4 \pi r^2$$

If the target does not absorb power and reradiates isotropically the return power, P_r, intercepted by the receiving antenna is:

$$P_r = \frac{P_t G}{4\pi r^2} \sigma_b \frac{1}{4\pi r^2} A_e$$

Where A_e is the effective aperture area of the antenna. From antenna theory, the effective area is related to the gain and wavelength, λ, by:

$$A_e = \frac{G\lambda^2}{4\pi}$$

Thus,

$$P_r = \left| \underbrace{\left\{ \overbrace{\left[\underbrace{\left(\frac{P_t G}{4 \pi r^2} \right)}_{\text{incident power density}} \sigma_b \right]}^{\text{backscattered power}} \frac{1}{4 \pi r^2} \right\}}_{\text{power density at receiving antenna}} \overbrace{\frac{G \lambda^2}{4 \pi}}^{\text{power collected by the antenna}} \right|$$

In the case of precipitation, the radar illuminates a large number of targets (raindrops) at the same time (Figure A-1) and the average return power is:

$$\overline{P}_r = \frac{P_t G^2 \lambda^2}{(4\pi)^3 r^4} \sum_i \sigma_{bi}$$

The summation is over the volume from which power is received simultaneously. This volume is proportional to the horizontal and vertical half-power beamwidths, θ_{3dB}, ϕ_{3dB}, and the radar sample volume depth, $c\tau/2$, i.e.:

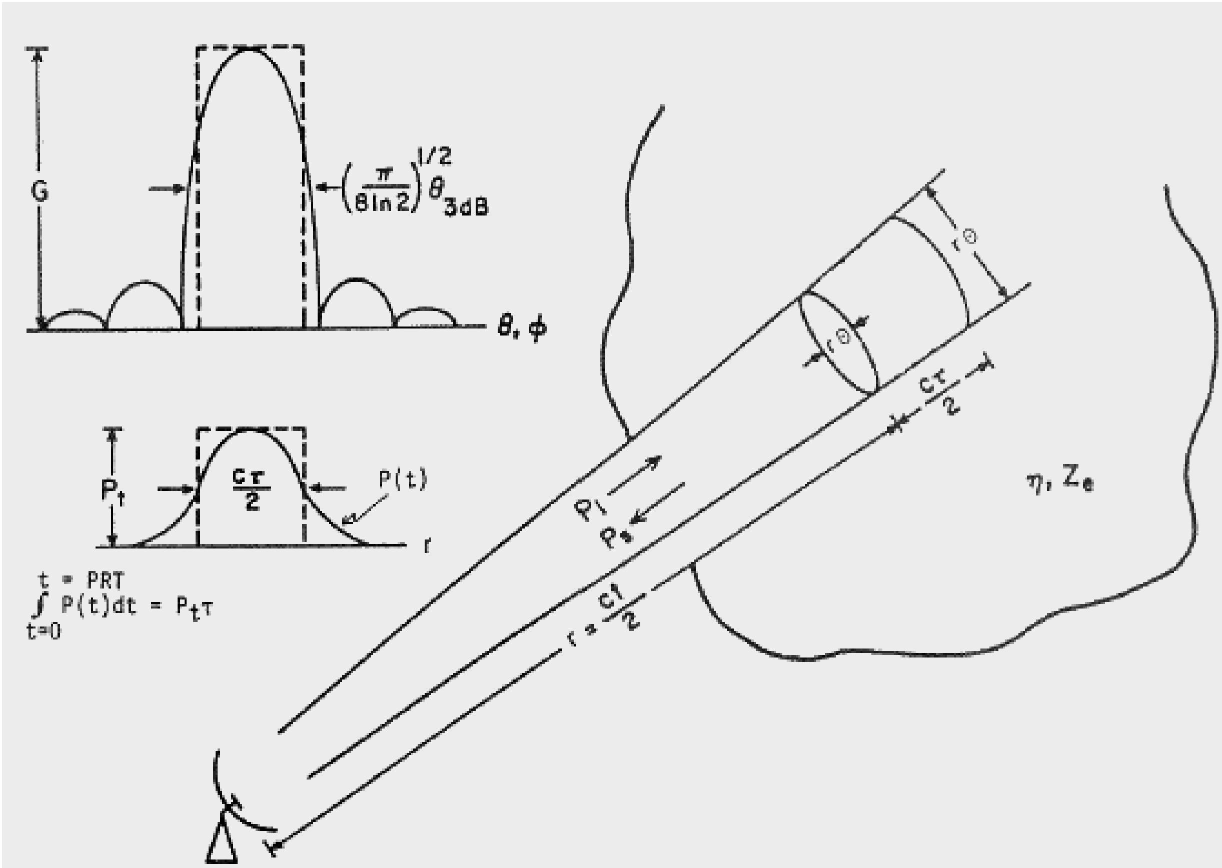

Figure A-1
Schematic for Radar Detection of a Distributed Target

The volume illuminated by the radar subtends an equivalent angle of $(\pi/8\ln2)^{1/2}$ times the antenna one-way 3 dB beamwidth (upper inset) and has a depth equal to the equi-energy pulse width that is very close to half the power width for the WSR-88D.

$$\mathrm{Vol} \propto \pi\left(\frac{\mathrm{r}\,\theta_{3dB}}{2}\right)\left(\frac{\mathrm{r}\,\phi_{3dB}}{2}\right)\frac{\mathrm{c}\,\tau}{2}$$

If targets are uniformly distributed over the volume, the total back-scattering cross section can be expressed as:

$$\sum_{i} \sigma_{bi} = \mathrm{Vol} \cdot \eta$$

where η is the backscattering cross section per unit volume.

The scattering volume angular dimension for circularly symmetric and Gaussian shape antenna patterns (Figure A-1, inset) is given by:

Equivalent Angle

$$(\theta_e, \phi_e) = \frac{\pi\theta_{3dB}^{2}}{8\ln 2}$$

The sample volume depth is given by the depth of the equivalent rectangular pulse containing the same energy as the actual transmitter pulse (Figure A-1, inset) that, for the pulse shape used in the WSR-88D, is very close to the pulse half-power width.

A.1.1 Radar Cross Section of Raindrops. The radar cross section of a spheroid exhibits the resonance property illustrated in Figure A-2. For incident wavelengths, λ, large, compared to the radius, a, the cross section is given by Rayleigh's law. In the opposite case, for λ small compared to the radius, the cross section approaches the geometric cross section. Between these limits are the resonance maxima or Mie region. For the WSR-88D wavelength of 10 cm and liquid drops of diameter less than about 10 mm, the backscattering is well described by the Rayleigh law. However, large hail and graupel could have dimensions that carry the scattering over to the Mie region.

In the case of Rayleigh scattering, the sphere becomes an electric dipole with a radar cross section that can be shown to be:

$$\sigma_b = 4\pi\left(\frac{2\pi}{\lambda}\right)a^6\left|\frac{\varepsilon - 1}{\varepsilon + 2}\right|^2$$

where ε is the dielectric constant. For a water drop, $|\varepsilon| \gg 1$, the above reduces to:

$$\sigma_b = 4\left(\frac{2\pi a}{\lambda}\right)^4 \pi a^2$$

For a conducting metal sphere with $\varepsilon = \infty$:

$$\sigma_b = 9\left(\frac{2\pi a}{\lambda}\right)^4 \pi a^2$$

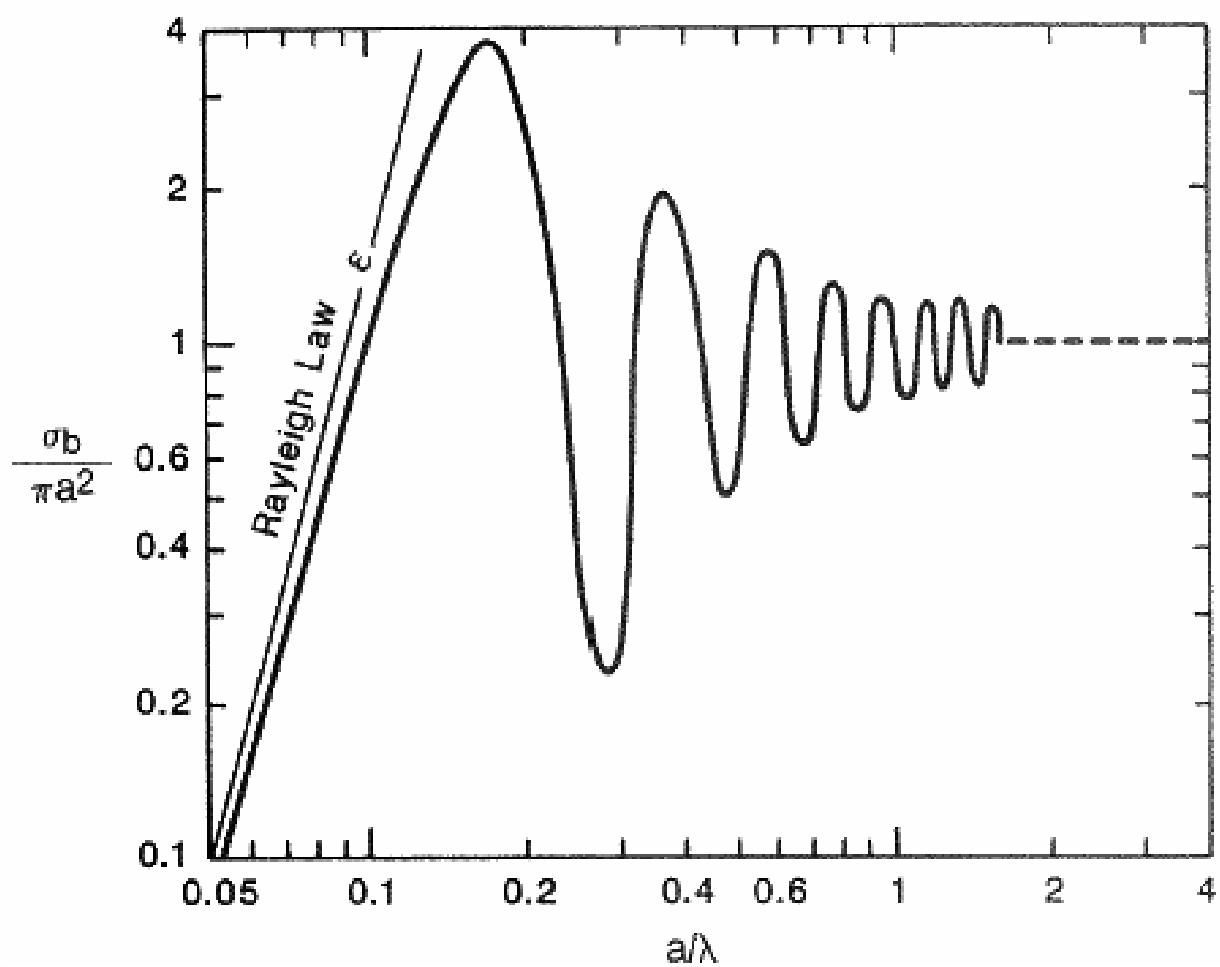

Figure A-2
Radar Cross Section of a Metallic Sphere

Radar cross section, σ_b, normalized to geometric cross section, πa^2. The straight line is the cross section predicted by the Rayleigh Law.

From the above, it is seen that σ_b is strongly dependent on the drop size ($\sigma_b \propto a^6$). The backscattering cross section per unit volume, η, is thus dependent on drop size distribution, i.e., the number of drops of radius a, $\eta(a)$. Since η is the summation of individual cross sections:

$$\eta = 4\pi \int_0^{\infty} a^2 \left(\frac{2\pi a}{\lambda} \right)^4 \eta(a)\, da$$

$$\eta = \frac{2^6 \pi^5}{\lambda^4} \int_0^{\infty} a^6 \eta(a)\, da$$

The integral in the above expression is, under suitable restriction, proportional to the total volume liquid water per unit volume, which is the quantity of interest with meteorological radars.

A.1.2 Equivalent Radar Reflectivity Factor. Total liquid water per unit volume is a more meaningful quantity to radar meteorologists than radar cross sections per unit volume and, by convention, radar meteorologists describe the meteorological target by the "equivalent radar reflectivity factor," Z_e, which is related to η by:

$$\eta = \frac{\pi^5}{\lambda^4} |K|^2 Z_e$$

where $|K|^2$ is the complex index of refraction (K = 0.93 for water and 0.2 for ice).

For example, if all drops have diameter D_e and radar cross section σ_b, Z_e reduces to:

$$Z_e = \frac{6\lambda^4 \sigma_b M}{\pi^6 |K|^2 \rho D_e^3}$$

where M is the mass liquid water content and ρ is the density of water.

Z_e is usually expressed in $mm^6\, m^{-3}$ requiring a units conversion factor of 10^{-18} to be consistent with the units of η, which is area per unit volume in m^{-1} in the standard equation.

A.1.3 Meteorological Radar Equation. Substituting the composite radar cross section:

$$\sum_i \sigma_{bi} = \mathrm{Vol} \bullet \eta = \frac{c\tau}{2} \frac{\pi\theta^2 r^2}{8\ln 2} \bullet \frac{\pi^5}{\lambda^4} |K|^2 Z_e$$

into the general radar equation we have:

$$\overline{P}_r = \frac{(\pi)^3 P_t G^2 \theta_{3dB}^2 \, c\tau |K|^2 Z_e}{2^{10} \lambda^2 r^2 \ln 2} L$$

where:

$\overline{P}_r$	= average return power, watts
P_t	= peak transmitted power, watts
G	= antenna gain, dimensionless
λ	= radar wavelength, meter
θ_{3dB}	= antenna half-power beamwidth, radian
τ	= pulse duration, second
c	= electromagnetic propagation constant $\simeq 3(10^8)$ m s^{-1}
r	= range to pulse volume, meter
K	= complex index of refraction; $\lvert K\rvert^2$ is conventionally taken to be 0.93 for water and 0.2 for ice.
Z_e	= equivalent radar reflectivity factor, m^3 (often expressed in $mm^6\ m^{-3}$ for use in empirical rainfall rate equations such as $Z = 200R^{1.6}$ with rate, R, in mm hr^{-1}).
L	= loss factors associated with propagation and receiver detection.

Typical detection capability of the WSR-88D is shown in Figure A-3. The ability to provide a quantitative measurement is not so dependent on return absolute power as it is on the signal to noise ratio.

A.2 Doppler Effect. Consider again the radar signal return as a function of time, t, from a single target at range r. At the receiver input this voltage, V, is given by:

$$V(t,\ r) = A \exp\left[j f\, 2\pi \left(t - 2\frac{r}{c} \right) + j \psi \right] U\left(t - 2\frac{r}{c} \right)$$

where A is the composite signal amplitude, f is the radar frequency, ψ is the initial phase and U is the gating function, U = 1, for $0<(t-2r/c)<\tau$, U = 0, otherwise.

After the first mixer:

$$V(t,\ r) = b\ A \exp\left[-j\left(\frac{4\pi r}{\lambda} - \psi \right) \exp(j\ 2\pi\ f\ \Delta t) \right] U\left(t - 2\frac{r}{c} \right)$$

where b is gain constant, the first exponential is the target modulation signal, and the second is the subcarrier.

Doppler radars capable of velocity sign detection, such as the WSR-88D, typically generate the signal phasor by homodyning down to zero frequency carrier both the return signal and the return signal phase shifted by $\pi/2$. The outputs are an Inphase, I, and Quadrature, Q, signal.

$$I(t,r) = A_I \cos\left(\frac{4\pi r}{\lambda} - \psi \right) U(t,r)$$

$$Q(t,r) = A_Q \sin\left(\frac{4\pi r}{\lambda} - \psi \right) U(t,r)$$

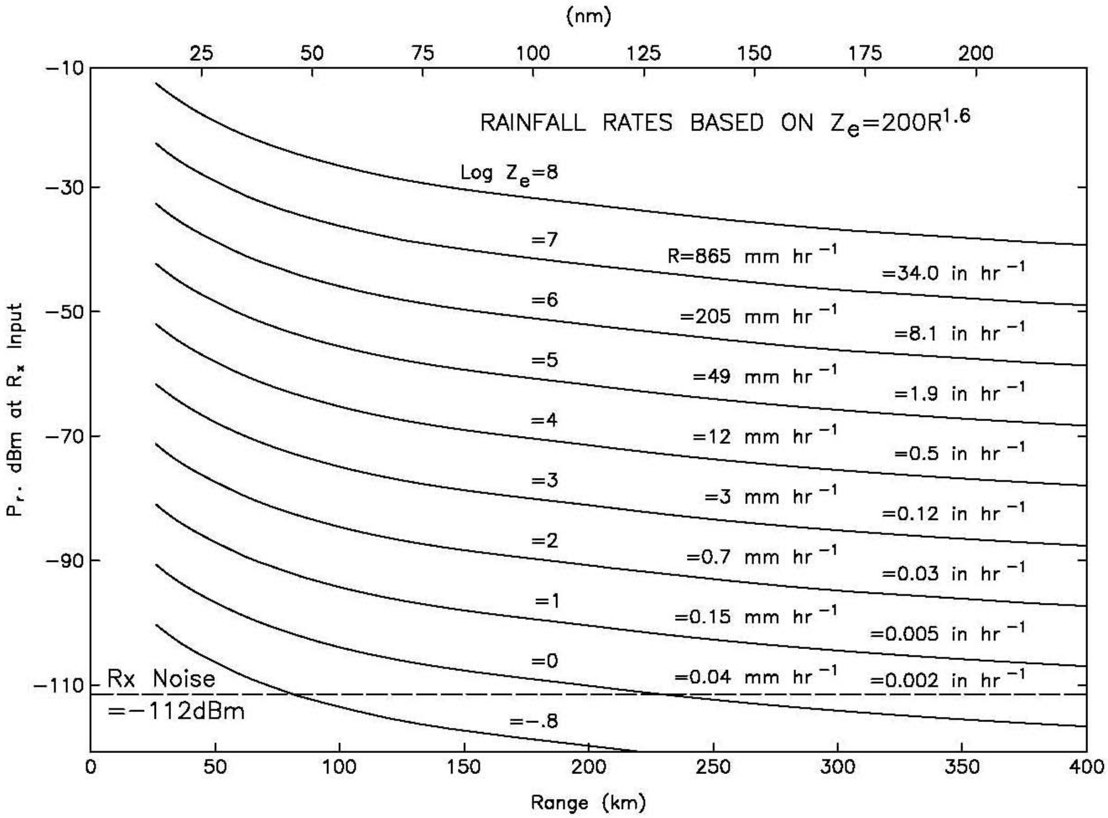

Figure A-3
Reflectivity Detection Capability of the WSR-88D

The equivalent reflectivity, Z_e, rainfall rate relationship of $Z_e = 200R^{1.6}$ is a good general relationship, but is not optimum for all types of liquid precipitation and is not valid for snow.

If the range, r, changes with time (target moving relative to radar with a velocity, v_r) the signal argument (phase) becomes a function of time.

Time rate of phase change is the angular velocity ω. Angular velocity is related to frequency, f, by $\omega = 2\pi f$, thus:

$$\frac{d}{dt}\left(\frac{4r\pi}{\lambda} - \psi\right) = \frac{4\pi}{\lambda}\frac{dr}{dt} = \frac{4\pi}{\lambda}v_r = 2f_d\pi$$

where f_d is the Doppler frequency and:

$$f_d = \frac{2v_r}{\lambda}$$

is the Doppler equation.

The frequency shift of interest in meteorological radars corresponds to radial velocities of tens of meters per second down to a fraction of a meter per second, resulting in very small phase shift during the radar pulse period.

For example, at $\lambda = 10$ cm the phase shift during a typical pulse width:

$$\tau \simeq 1.4\ (10^{-6})\ \text{s}$$

for a radial velocity, v_r, of 1 m s^{-1}, is:

$$\Delta\ \text{angle} = \left(\frac{4\pi}{\lambda}v_r\right)\Delta t \approx 10^{-2}\,\text{deg}$$

Such small phase shifts cannot be measured with existing techniques. Consequently most radars are configured to measure phase shift from pulse to pulse (PRT ~ 10^{-3} s), which is a much larger angle; about 1 degree for the above example.

However, this results in a sample data system subject to the Nyquist sampling criteria:

$$f_n = \frac{f_s}{2}$$

where f_n is the maximum unambiguous frequency (Nyquist frequency) and f_s is the sampling frequency (radar pulse repetition frequency).

Since the radar unambiguous range is also dictated by the pulse repetition frequency, there is a coupling between maximum unambiguous frequency, f_n (or velocity, v_a), and unambiguous range, r_a, given by:

$$f = \frac{f_s}{2} = \frac{2v_a}{\lambda} = \frac{1}{2r_a}\frac{c}{2}$$

and:

$$v_a r_a = \frac{c\lambda}{8}$$

This relationship, shown in Figure A-4, is one of the more important constraints on pulse Doppler radar, and gives rise to the operational problems of range folding and velocity aliasing.

A.3 <u>Statistics of the Raindrop Array</u>. As previously noted, the weather echo is not produced by a single target but is made up of signals from a dense array of point scatterers and the instantaneous signal is a vector sum of amplitude and phase of weighted returns:

$$V(t,\ r) = \sum_i A_i \exp\left(j \frac{4\pi r_i}{\lambda} \right)$$

Instantaneous echo sample power is proportional to the product of voltage and its complex conjugate:

$$P \propto \frac{1}{2}[VV^*] = \frac{1}{2}\sum_{i,\,k} A_i A_k^* \exp\left(j\, 4\, \pi \frac{(r_i - r_k)}{\lambda} \right)$$

therefore,

$$P \propto \frac{1}{2}\sum_i A_i^2 + \frac{1}{2}\sum_{i \neq k} A_i A_k^* \exp\left(j\, 4\, \pi \frac{(r_i - r_k)}{\lambda} \right)$$

Under constraints of long-term averaging, uniform distribution of targets, and random shuffling of targets within the radar sample volume, the second term tends to zero. Return power can then be related to the sum of the scatterers, $\sum_i A_i^2$, and interactive effects of the raindrop array ignored.

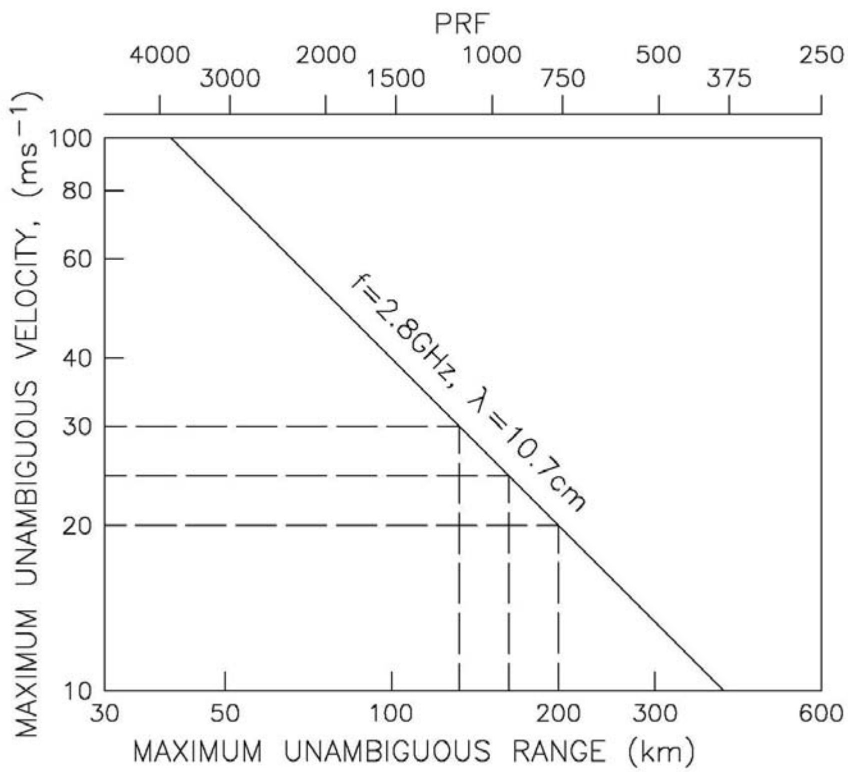

Figure A-4
Unambiguous Range-Velocity Relationship for the WSR-88D

Dashed lines are for unambiguous velocities of 30, 25, and 20 ms^{-1} with associated unambiguous ranges of 134, 160, and 200 km.

An important statistical parameter can be inferred from the statistical independence of the individual signal contributions. If the I, Q signals are composed of a large number of statistically independent contributions, the amplitude probability densities approach a Gaussian function by the Central Limit Theorem and (σ is standard deviation):

$$\text{Prob } A_I = \frac{1}{2\pi\sigma^2} \exp\left(- \frac{I^2}{2\sigma^2} \right)$$

$$\text{Prob } A_Q = \frac{1}{2\pi\sigma^2} \exp\left(- \frac{Q^2}{2\sigma^2} \right)$$

The probability of any function of these signals such as signal return power, $P_r = A_I + A_Q$, signal envelope amplitude, $A = [A_I + A_Q]^{1/2}$, or the logarithm of the return power $\text{Log } P_r = \text{Log } [A_I + A_Q]$ can then be derived by probability density transformation. However, in WSR-88D signal processing, the return power estimates are made directly from averages of return power rather than indirectly from signal amplitude or log power; thus, the statistics of these functions of power are of little interest.

Whereas the signal amplitude and power statistics describe the meteorological target backscattering cross section, the frequency or velocity statistics describe target motion, weighted by the target reflectivity. The function of velocity, most easily interpreted in terms of the meteorology, is the power spectral density. The spectral density used here is the velocity spectral density, i.e., the return power as a function of velocity (velocity being related to frequency by the Doppler equation). Parameters of interest are the first moment about zero or mean velocity and first moment about the mean or spectrum width.

The mean velocity thus represents a radar return, power-weighted mean of all scatterers in the "radar sample volume." Spectrum width is a measure of the dispersion of scatterer velocity about this mean.

Spectrum width is a function of radar system characteristics such as beamwidth, pulse width, wavelength and antenna rotation rate, as well as meteorological parameters describing the velocity dispersion within the radar sample volume. Some of the more significant contributions to velocity variance, σ_v^2, are:

$$\sigma_v^2 = \sigma_d^2 + \sigma_r^2 + \sigma_s^2 + \sigma_t^2$$

Where:

σ_d^2 = variance due to drop size distribution

σ_r^2 = variance due to antenna motion

σ_s^2 = variance due to wind shear
σ_t^2 = variance due to turbulence

If none of the mechanisms are dominant, we would expect the spectral density to be Gaussian by the Central Limit Theorem. With rare but important exception (mesocyclone and tornadic vortex), this is the case.

An estimate of the contribution due to individual effects can be made as follows:

Drop Size Distribution

$\sigma_d^2 = (\sigma_{do} \sin \phi_e)^2$

σ_{do} = standard deviation of drop terminal velocities ($\simeq$ 1 m s^{-1})

ϕ_e = antenna elevation angle

Antenna Rotation

$$\sigma_r^2 \sim \left(\frac{\alpha \lambda}{10.7 \theta_2} \right)$$

where:

α = antenna rotation rate, deg s^{-1}
λ = wavelength, m
θ_2 = two-way antenna 3 dB beamwidth, deg
($\theta_2 \simeq$ 0.7 θ_{3dB} for Gaussian beams)

Wind Shear

$$\sigma_s^2 = \sigma_{s\theta}^2 + \sigma_{s\phi}^2 + \sigma_{sr}^2$$

for constant gradients, k, in azimuth, elevation, and range and a Gaussian antenna pattern

$$\sigma_{s\theta}^2 + \sigma_{s\phi}^2 = (r\, \sigma_\theta\, \kappa_\theta)^2 + (r\, \sigma_\phi\, \kappa_\theta)^2$$

$$\sigma_{sr}^2 = [\beta\, k_r\, (c\,\tau/2)]^2$$

where:

1) κ is a shear constant (meters per second per meter)

2) $\sqrt{s\theta}^2$ is variance due to wind gradient in the azmuthal direction

3) $\sqrt{s\phi}^2$ is variance due to wind gradient in elevation

4) $\sqrt{s\,r}^2$ is variance due to wind gradient in range

$\beta \simeq 0.29$ for rectangular pulse and wideband receiver

$\beta \simeq 0.34$ for rectangular pulse and matched receiver

Turbulence – Median Values:

$$2m^2s^{-2} \leq \sigma_t^2 \leq 16m^2s^{-2}$$

Stratiform rain snow — convective storms

Typical composite values are shown in Figure A-5 for:

λ = 10 cm
θ = 1°
α = 18° s^{-1}
κ = 4 (10^{-3}) s^{-1}
ϕ_t^2 = 1.4 m s^{-1}

Note the range dependency of σ_V predicted if the shear is sustained over the radar sample volume at all ranges. In practice, this is usually not found to be the case. Small scale turbulence is the more significant contribution, especially in convective storms.

A.4 <u>Return Power, Velocity, and Spectrum Width Estimation</u>. The uncertainty associated with a single sample of return power, amplitude, or spectrum width is very large. Consequently, the estimate consist of averages where the standard deviation is reduced to an acceptable value, typically 1 dBZ_e for return power and 1 m s^{-1} for the mean velocity and spectrum width.

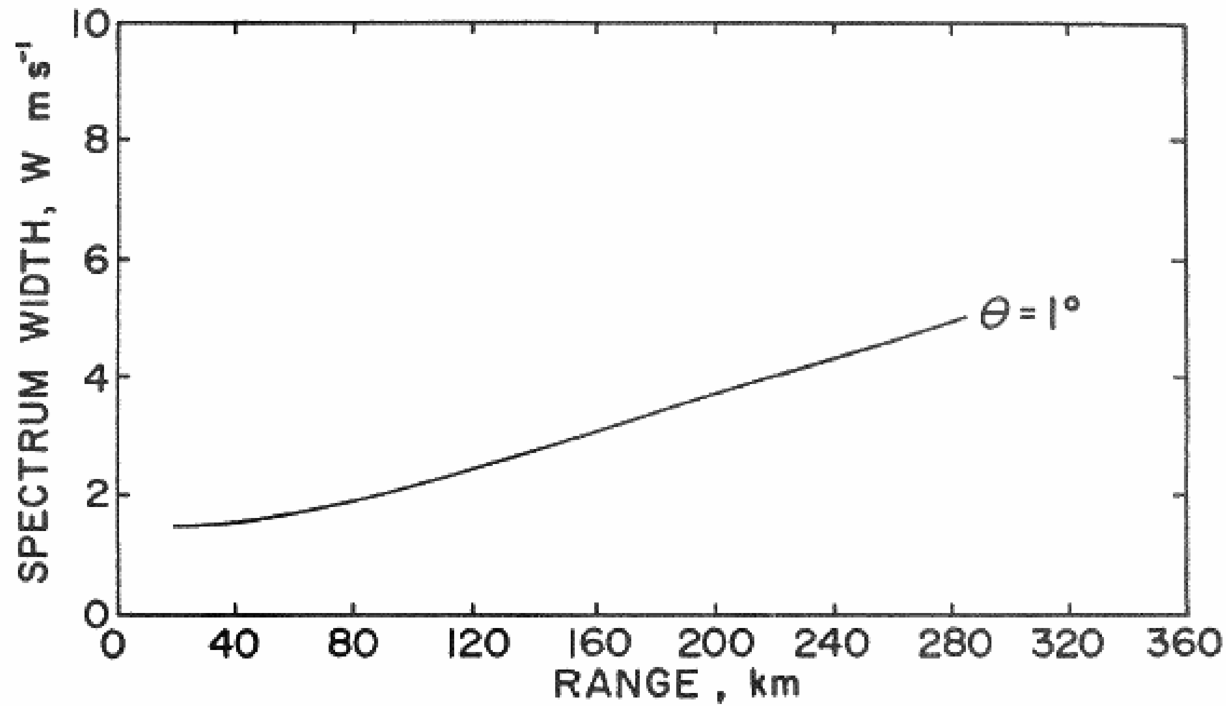

Figure A-5
Theoretical Spectrum Width Versus Range for the WSR-88D

Spectrum width at an antenna speed of 3 rpm assuming a vertical wind shear of $4(10^{-3})\ s^{-1}$ and a small turbulence of $1.4\ ms^{-1}$.

A.4.1 Estimation of Return Power (Z_e). Return power is estimated from the signal envelope by a combination of time (pulse to pulse) and range (adjacent sample volume) averaging.

A mean estimate, $\overline{X}$, by a linear average of N independent samples drawn from a population with variance σ_i^2:

$$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

has variance,

$$\sigma\overline{X}^2 = \frac{1}{N}\sigma_i^2$$

However, if the samples are correlated, the variance of a Ns sample average becomes:

$$\sigma\overline{X}^2 = \sigma_i^2 \sum_{m=-(Ns-1)}^{Ns-1} \frac{Ns - |m|}{Ns^2} R(mTs) \quad = \frac{1}{N_I}\sigma_i^2$$

where R(mTs) is the correlation between samples and Ts is the sampling interval. The mean estimate variance ratio for N_I independent samples to Ns correlated samples is:

$$\frac{\sigma I^2}{\sigma s^2} = \frac{Ns}{N_I} = \sum_{m=-(Ns-1)}^{Ns-1} \left(1 - \frac{|m|}{Ns}\right) R(mTs)$$

where:

$$N \sim \frac{4\sqrt{\pi}\,\sigma v\ Ns\ Ts}{\lambda}$$

The correlation of the samples is related to the PRT and the velocity spectrum width by the following. In general the spectral density functional form is Gaussian:

$$W(f) = W_o \exp\left[-\frac{(f - \bar{f})^2}{2\sigma_f^2}\right]$$

where σ_f is the frequency standard deviation.

The normalized autocorrelation is given by the inverse Fourier transform:

$$r(t) = \exp\left[-\frac{t^2}{2\sigma_t^2}\right]$$

where σ_t is the time standard deviation.

Parameters σ_f and σ_t are related to each other by:

$$\sigma_t = \frac{1}{2\pi\sigma_f}$$

and to the velocity spectrum by:

$$\sigma_f = \frac{2\sigma_v}{\lambda}$$

where σ_v is the standard deviation of the velocity spectrum, i.e.:

$$W(v) = W \exp\left[-\frac{(v - \overline{v})^2}{2\sigma_v^2}\right]$$

and:

$$r(t) = \exp\left[-t^2 \frac{2\pi\,\sigma_v^2}{\lambda}\right]$$

For a receiver having an output directly proportional to input power (square law receiver) used in the WSR-88D, the amplitude variance is twice the frequency prior to amplitude detection and the correlation becomes:

$$r(t) = \exp\left[-t^2 \frac{2\pi\,\sigma_v^2}{\lambda}\right]$$

Estimate variance can also be reduced by averaging in range. The averaging interval is chosen so as to preserve the small-scale features of the meteorology and range sample spacing is selected by consideration of signal redundancy from correlation by transmitter pulse and receiver band width, and reduction of noise variance at low signal to noise ratio.

As the discrete average approaches a continuous integration, the number of independent samples in range approaches:

$$N_{Ir} = \frac{3m}{2} + \frac{3}{8},$$

where m is the number of pulse depths averaged.

The overall estimate variance reduction factor is the product of the number of independent samples in range and the number of independent samples in time with the product usually being about fifty.

A.4.2 Estimation of Mean Velocity. Mean velocity estimates in the WSR-88D system are made by a technique that circumvents the spectral density calculation and estimates the first moment of the spectral density from the argument of the complex covariance. The technique is commonly referred to as "pulse-pair processing."

The rigor of this estimator lies in the Moment Theorem. The moments of a random variable, w:

$$E\,(w^{n}) = M_{n}$$

are related to the derivatives of its characteristic function, Φ , by:

$$jM = \frac{d^{n\Phi}(o)}{dx^{n}}$$

In particular this implies that, since the complex covariance and the spectral density constitute a Fourier transform pair, the spectral density moments correspond to the complex covariance evaluated at zero lag.

Expressing the covariance in polar form:

$$r(\tau) = A(\tau)\exp[\,j\,2\pi\,\delta(\tau)]$$

Where $A(\tau)$ is a real even function of τ and $\delta(\tau)$ is a real odd function of:

$$f = \frac{1}{j2\pi}\frac{\frac{d}{d\tau}[r(\tau)]}{r(\tau)}\bigg|_{\tau=0} = \frac{d}{d\tau}[\delta(\tau)]\bigg|_{\tau=0}$$

that for small $\tau_s = 0$:

$$\frac{d}{d\tau}[\delta(\tau)]\;\frac{\delta(\tau_s) - \delta(0)}{\tau_s} = \frac{\delta(\tau_s)}{\tau_s}$$

and:

$$\hat{f} = \frac{1}{2\pi\tau_s}\operatorname{Arg}\left[r(\tau_s)\right]$$

A maximum likelihood unbiased estimator of r(τ):

$$r(\tau_s) = \frac{1}{N} \sum_{n=1}^{N} Z_{n+1} Z_n^*$$

where:

$Z \quad = \quad I + jQ$
$Z^* \quad = \quad$ complex conjugate of Z

forms the basis for an estimator of spectral density mean given by:

$$f_{ppp} = \frac{1}{\pi} \arctan \frac{\sum_n \operatorname{Im}\left(Z_{n+1} Z_n^*\right)}{\sum_n \operatorname{Re}\left(Z_{n+1} Z_n^*\right)}$$

The restrictions imposed on A(t) and w(t) require the spectral to be unimodal and symmetric about its mean, This is generally true for meteorological signals but there are important exceptions such as the mesocyclone and the tornadic vortex. Performance of this estimator is shown in Figure A-6.

A.4.3 Estimation of Spectrum Width. Spectrum width estimates in the WSR-88D unit are also calculated in the time domain. The estimate is by signal autocorrelation and the rigor is derived from the fact that signal correlation and spectral density constitute a Fourier Transform Pair. The particular algorithm used, however, is valid only for weather signals having Gaussian spectra.

Fundamentally, the width frequency estimate is the standard deviation of the input spectral density (assumed to be Gaussian) given by the autocorrelation of input signal, i.e.:

$$W = \frac{1}{\sqrt{2}\pi \tau_s} \left| \ln \left(\frac{\hat{S}}{|\hat{r}(\tau_s)|} \right) \right|^{1/2} \text{Hz}$$

where τ_s is the sampling interval (PRT), S is the signal power estimate after removal of noise power from total power, and $r(\tau_s)$ is the complex covariance at lag τ_s.

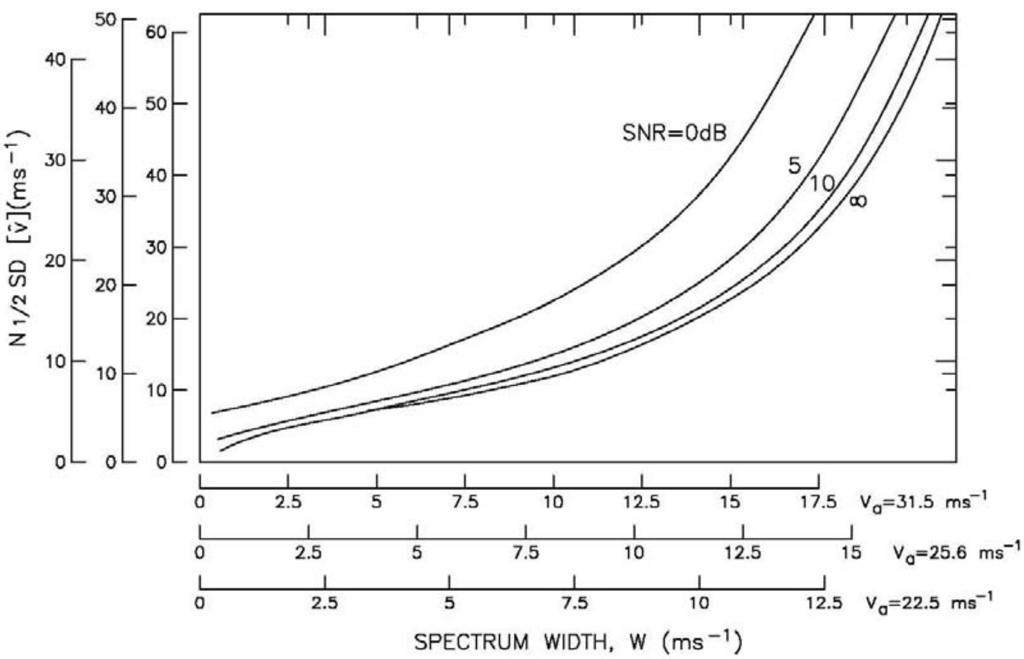

Figure A-6
Standard Deviation of the Mean Velocity Estimate

This figure depicts the normalized standard deviation of the mean velocity estimate as a function of spectrum width for three unambiguous velocities and four levels of signal-to-noise ratio (SNR). Note the ordinate value must be divided by $N^{1/2}$, the square root of the number of samples in the estimate.

The algorithm used delivers the width velocity and uses intermediate terms of the covariance as well as logarithmic algebra to be expressed as

$$\hat{W} = \frac{v_a}{\pi}\left[\left|\ln\left(\frac{\dfrac{R_e^2 + Im^2}{(m-1)^2}}{\left(\dfrac{Pn}{m} - n\right)^2}\right)\right|\right]^{1/2} \quad m\ s^{-1}$$

where:

- v_a = Unambiguous velocity
- R_e = real part of $R(\tau_s)$
- Im = imaginary part of $R(\tau_s)$
- Pn = signal plus noise power
- n = noise power
- m = number of samples in the estimate

The performance estimator is shown in Figure A-7.

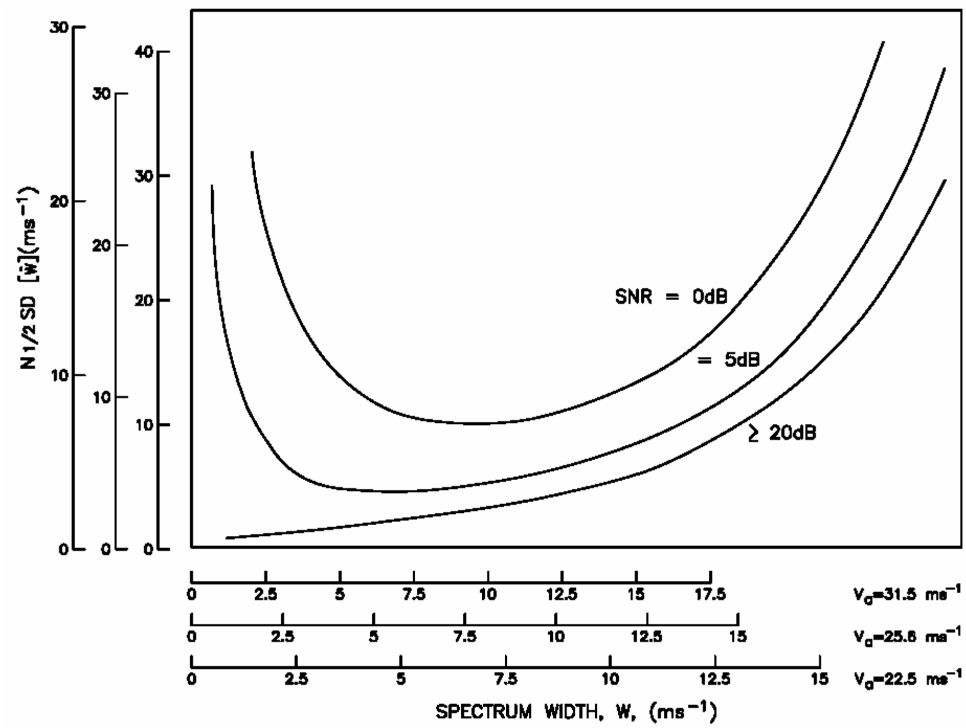

Figure A-7
Standard Deviation of the Spectrum Width Estimate

This figure depicts the normalized standard deviation of the spectrum width estimate as a function of spectrum width for three unambiguous velocities and three levels of signal-to-noise ratio (SNR). Note the ordinate value must be divided by $N^{1/2}$, the square root of the number of samples in the estimate.

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